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NUMERICAL METHODS AND APPROXIMATION AND MODELLING PROBLEMS IN S--ETC(U)
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FINAL REPORT

LEVEL II

NUMERICAL METHODS AND APPROXIMATION AND MODELLING PROBLEMS IN STOCHASTIC CONTROL THEORY

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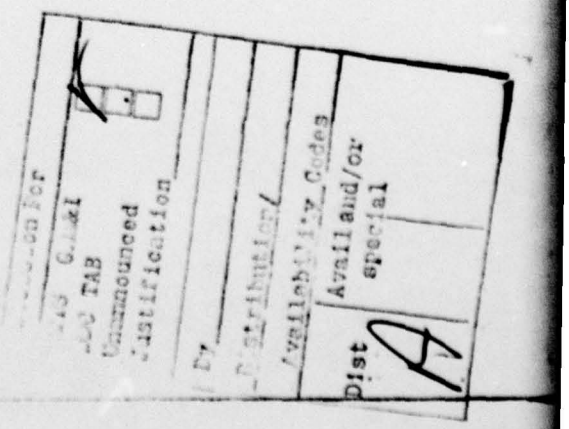
August 15, 1979

ABSTRACT

Many areas of stochastic control were studied in this period. Monte-Carlo algorithms of type appearing in adaptive systems were studied and stability, convergence and rate of convergence results obtained. The analytical techniques developed for this purpose have a wide applicability in the study of similar algorithms. Robust computationally oriented approximations to optimal non-linear filters were developed. A deep study of how to find the Markov system that best approximates a given non-linear system with a wide band -width input was undertaken and very good results obtained. The problem is important in control and communication theory because it is usually desired to use the many available methods for analyzing Markov systems. The results obtained seem to be the best available so far and applications to control and communication theory are being pursued. Optimal stochastic control problems under the condition of partial information on the system state were studied and a new formulation, similar to that of the non-linear filtering problem, developed. The approach seems promising in providing qualitative information on this difficult class of problems. Part of the technique uses measure-valued processes in a way which has wider applicability to problems involving distributed states or information.



SUMMARY

H. J. Kushner continued his investigations into numerical and recursive monte-carlo methods. One particularly important result is [2], where both convergence and rate of convergence and stability results are developed for stochastic approximations of the type $X_{n+1} = X_n + ah(X_n, \xi_n)$, for small a, where $\{\xi_n\}$ is a rather arbitrarily correlated noise sequence. This type of algorithm occurs, or is heavily used, in many adaptive control and communication systems, adaptive filters and identification methods. The results are the first of their type and shed considerable light on these procedures under realistic conditions. In this work the techniques of [1] were extended, to cover the practically important cases dealt with in [2]. Many new ideas and applications are currently being developed.

The problem of robustness of non-linear filters is an old and important one, but it is only recently that significant results have become available. Reference [5] by Kushner developes an approximately optimal computational method for non-linear filtering. This filter has particularly nice properties in that the estimate is a continuous function in the observational data, uniformly in the approximation parameter. This smoothness is particularly important in applications, since it is important that the filter not be oversensitive to modelling assumptions.

A particularly interesting and practically important area of research was initiated by Kushner [4], [5]. These references deal with the problem of finding the closest 'nice' Markov system to a particular given nonlinear stochastic system, with wide

band -width noise inputs. These problems occur frequently in both control and communication theory, and it is quite common, often using purely formal methods, to try to find a nice Markov approximation, so that the powerful associated analytical methods can be used. References [4], [5] give the best currently available methods and conditions. The techniques are also simpler than those used in past works.

Some of the numerous applications to control and communication theory are being investigated and, so far, the results are very nice. This latter work is being done with Professor Y. Bar-Ness of Tel Aviv University who is visiting us this year.

References

[1] H.J. Kushner, Huang Hai, "Rates of Convergence for stochastic approximation type algorithms", to appear SIAM J. on Control.

[2] H.J. Kushner, Huang Hai, "Asymptotic properties of stochastic approximations with constant coefficients", submitted to SIAM J. on Control.

[3] H.J. Kushner, "Jump-diffusion approximations for ordinary differential equations with wide bound random right hand sides", to appear SIAM J. on Control.

[4] H.J. Kushner, "Approximation of solutions to differential equations with random inputs by diffusion processes", Bonn conference on stochastic control, to appear in Springer Lecture Notes in Operations Research and Economics.

[5] H.J. Kushner, "A robust computable approximation to the optimal non-linear filter", to appear, Stochastics.

W. H. Fleming continued his program of research in optimal stochastic control, with complete or partial observations, and on Markov diffusion models of distributed parameter stochastic systems. A new direction of work is the study of optimal control of partially observed stochastic systems using techniques of measure-valued processes. This is done by introducing a new "separated" stochastic control problem with completely observed states, the states in the new problem being probability measures on the original state space. The original and separated control problems are related by the nonlinear filter equation for the conditional state distribution given the observed data. By understanding well the relation between these two control problems, it is hoped to shed light on some basic theoretical questions of long standing. One of these questions is to give appropriate conditions for the existence of optimal controls in case of partial observations.

Papers Published or to appear

[1] W.H. Fleming, Optimal control of Markov diffusion processes, Proc. Joint Automatic Control Conf., Oct. 1978.

[2] W.H. Fleming, Large deviations for diffusions depending on small parameters: a stochastic control method, Proc. 1st AFCET-SMF Symposium, École Polytechnique, France, Sept. 1978.

[3] W.H. Fleming, Exit probabilities for diffusions depending on small parameters, Proc. Oxford Symposium, Sept. 1978.

[4] W.H. Fleming and M. Viot, Some measure-valued processes in population genetics theory, to appear in Indiana J. of Math.

[5] W.H. Fleming, Measure-valued processes in the control of partially observed stochastic systems, to appear in Applied Math. and Optimization (presented at UCLA Systems Science Workshop, March 1979.)

Martin Day visited at Brown during February - June 1979. He recently found a significant improvement of Fleming's stochastic control method (J. Applied Math. Optimization 1978, pp.329-346) for estimating exit probabilities for diffusions depending on small parameters. A report of this work is in preparation. An extension of the method to study the asymptotics as $\varepsilon \to 0$ of solutions to the partial differential equation $\varepsilon \Delta u + b(x) \cdot \nabla u = 0$ with Dirichlet boundary data is currently being tried by Day and Fleming.

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